a significantly greater reduction in noise level than could be accounted for by their equivalent absorption area alone.

II.3.3 Reverberation in Coupled Rooms

We now proceed to non-stationary processes in coupled rooms; thus, we must drop the power balances given in eqns. (3.11) and (3.12). Instead, we assume that the introduction and removal of energy in both rooms leads to time-varying changes in the total energy contents, (E_1V_1) and (E_2V_2), of the two rooms. We represent the stopping of the second source by setting $P_1=0$ and attend to the reverberant sound decay only. Thus, instead of eqns. (3.11) and (3.12), we have the differential equations:

$$\frac{c}{4}(A_{11}E_1-\tau S_{12}E_2)=-V_1\frac{dE_1}{dt} \tag{3.18}$$

$$\frac{c}{4}(-\tau S_{12}E_1+A_{22}E_2)=-V_2\frac{dE_2}{dt} \tag{3.19}$$

Since these equations are linear, we set

$$E_{12}=E_{01,2}e^{-2\delta t} \tag{3.20}$$

which means that we assume that the reverberant process is made up of exponentially decaying functions. The quantity δ which characterizes the rate of decay of the sound pressure is called the damping constant. Since we must deal here with energies, which are proportional to the squares of the sound pressures, the quantity 2δ appears in the exponent. Between δ and Sabine's reverberation time, the following relations hold:

$$2\delta=\frac{6}{T}\ln 10; \qquad \delta=\frac{6{\cdot}9}{T} \tag{3.21}$$

By setting eqn. (3.20) into eqns. (3.18) and (3.19), and omitting the common factor $e^{-2\delta t}$, we get for E_1 and E_2, and so also for their initial values E_{01} and E_{02}, the linear equations:

$$\left(\frac{c}{4}A_{11}-2\delta V_1\right)E_{01}-\frac{c}{4}\tau S_{12}E_{02}=0 \tag{3.22}$$

$$-\frac{c}{4}\tau S_{12}E_{01}+\left(\frac{c}{4}A_{22}-2\delta V_2\right)E_{02}=0 \tag{3.23}$$

These two equations can be valid for a simple exponential decay with a single value of δ *only* if both equations result in the same ratio for E_{01}/E_{02}. But this requires that the determinant of the coefficients of E_{01} and E_{02} must vanish:

$$\begin{vmatrix} \left(\frac{c}{4}A_{11}-2\delta V_1\right) & -\frac{c}{4}\tau S_{12} \\ -\frac{c}{4}\tau S_{12} & \left(\frac{c}{4}A_{22}-2\delta V_2\right) \end{vmatrix} = 0 \tag{3.24}$$

The resulting so-called 'characteristic equation' for δ becomes simpler if we introduce the damping constants:

$$\delta_1 = \frac{cA_{11}}{8V_1}, \qquad \delta_2 = \frac{cA_{22}}{8V_2} \tag{3.25}$$

which correspond to the decays of the two rooms as they would be if the rooms were uncoupled and the quantity $\alpha_{1,2}S_{12}$ were included in their respective absorption areas.

Equation (3.24) may then be written:

$$\left(1-\frac{\delta}{\delta_1}\right)\left(1-\frac{\delta}{\delta_2}\right)-k_1k_2=0 \tag{3.26}$$

where k_1 and k_2 are the coupling factors according to eqns. (3.8) and (3.10), but multiplied by τ. Since only their product appears in eqn. (3.26), we may introduce their geometric mean

$$\kappa = \sqrt{k_1k_2} \tag{3.27}$$

and call it the mean coupling coefficient, as is usual in the theory of coupled oscillators.

Since eqn. (3.26) is quadratic in δ, we must expect two different damping constants; this is not surprising, since even uncoupled rooms would have two different values of δ in general. The corresponding eigenvalues δ_{I} and δ_{II} for the coupled rooms are:

$$\delta_{\mathrm{I,II}} = \tfrac{1}{2}(\delta_1+\delta_2) \mp \sqrt{\tfrac{1}{4}(\delta_1-\delta_2)^2+\kappa^2\delta_1\delta_2} \tag{3.28}$$

The difference between δ_{I} and δ_{II} is greater, the greater the coupling coefficient κ. If we assume $\delta_1<\delta_2$, we get:

$$\delta_{\mathrm{I}}<\delta_1<\delta_2<\delta_{\mathrm{II}}$$

With $\kappa \to 0$, δ_{I} approaches δ_1 from below, and δ_{II} approaches δ_2 from above.

If the coupling between rooms is provided by a partition wall or a door, the value of κ becomes so small that the differences $(\delta_1 - \delta_{\mathrm{I}})$ and $(\delta_{\mathrm{II}} - \delta_2)$ become negligible. In this case, we can neglect not only τS_{12} but even $\alpha_{1,2} S_{12}$ in the equations:

$$A_{11} = A_{10} + \alpha_1 S_{12}; \qquad A_{22} = A_{20} + \alpha_2 S_{12} \tag{3.29}$$

Thus, the damping constants δ_1 and δ_2 are hardly any different from:

$$\delta_{10} = \frac{cA_{10}}{8V_1}, \qquad \delta_{20} = \frac{cA_{20}}{8V_2} \tag{3.30}$$

which we found for impenetrable coupling surfaces.

If small rooms are coupled to a large room by open areas $(\tau = 1)$ (which was our original problem, see Fig. 3.1), κ is generally very small. Even if $k_2 = S_{12}/A_{22}$ is nearly unity, at least $k_1 = S_{12}/A_{11}$ will be very small because of the large value of A_{11}. For small κ, the differences between δ_{I} and δ_1, and between δ_{II} and δ_2 are always small. They are greatest for $\delta_1 = \delta_2$, but even then they are not larger than $\kappa\delta_1$. But if δ_1 and δ_2 themselves differ greatly—or, more precisely, if

$$\frac{(\delta_1 - \delta_2)^2}{4\delta_1\delta_2} = \frac{1}{4}\left(\sqrt{\frac{\delta_1}{\delta_2}} - \sqrt{\frac{\delta_2}{\delta_1}}\right)^2 \gg \kappa^2 \tag{3.31}$$

(a condition which is fulfilled for coupled theater boxes on account of the different volumes), then we may express the square root in eqn. (3.28) in a power series in (κ^2), neglect all but the first term, and get:

$$\delta_{\mathrm{I}} = \delta_1 - \kappa^2 \frac{\delta_1\delta_2}{\delta_2 - \delta_1}$$

$$\delta_{\mathrm{II}} = \delta_2 + \kappa^2 \frac{\delta_1\delta_2}{\delta_2 - \delta_1} \tag{3.32}$$

The presupposition (eqn. (3.31)) for this development shows that the correction terms are so small, compared with $\sqrt{\delta_1\delta_2}$, that in practice the damping constants of the given uncoupled rooms could be used as a good approximation. By 'given' we mean that S_{12} (or $\alpha_{1,2}S_{12}$) must be added to the other absorption areas, A_{10} and A_{20}, of the rooms. But this is just what we found to be expedient in our discussion of the steady-state condition.

A high degree of coupling is possible in room acoustics only if the partial absorption areas, A_{10} and A_{20}, are small compared to the coupling area: this condition would, therefore, begin to approach that of a single, rather reverberant room. In such a case, it is more suitable to express eqn. (3.28) in the form:

$$\delta_{\mathrm{I,II}}=\frac{1}{2}(\delta_1+\delta_2)\mp\sqrt{\frac{1}{4}(\delta_1+\delta_2)^2-(1-\kappa^2)\delta_1\delta_2} \tag{3.33}$$

and to develop the square root in a power series in $(1-\kappa^2)$. This results in:

$$\delta_{\mathrm{I}}\approx(1-\kappa^2)\frac{\delta_1\delta_2}{\delta_1+\delta_2} \tag{3.34}$$

and:

$$\delta_{\mathrm{II}}\approx\delta_1-\delta_2 \tag{3.35}$$

With $A_{10}\ll S_{12}$ and $A_{20}\ll S_{12}$, eqn. (3.34) approaches

$$\delta_{\mathrm{I}}\approx\frac{c}{8}\frac{A_{10}+A_{20}}{V_1+V_2} \tag{3.36}$$

This means that we get a damping constant corresponding to a single room having an equivalent absorption area of $(A_{10}+A_{20})$ and a volume of (V_1+V_2).

II.3.4 Examples of Reverberation in Coupled Rooms

For loosely coupled rooms (that is, for small κ), the damping constants are practically the same as for the uncoupled rooms (with $\alpha_{1,2}S_{12}$ included in their respective absorption areas); but this does *not* mean that the reverberation process is the same as when there is no coupling. Even for small τ, we may hear in the neighboring room (if we hear anything at all through the partition) the initial portion of the reverberation in the source room. Moreover, the decay corresponding to the neighboring room enters into the resulting reverberation in both rooms.

This resulting reverberation comprises in both rooms terms proportional to $e^{-2\delta_{\mathrm{I}}t}$ and $e^{-2\delta_{\mathrm{II}}t}$:

$$E_1=E_{\mathrm{I}1}e^{-2\delta_{\mathrm{I}}t}+E_{\mathrm{II}1}e^{-2\delta_{\mathrm{II}}t} \tag{3.37}$$

$$E_2=E_{\mathrm{I}2}e^{-2\delta_{\mathrm{I}}t}+E_{\mathrm{II}2}e^{-2\delta_{\mathrm{II}}t} \tag{3.38}$$

where the quantities E_{I1}, etc., refer to the initial values for the different exponential decays; they may even be negative. (For brevity here, we drop the additional subscript 0, used to signify the time $t=0$.) In both rooms, the reverberation ends with the exponential function having the smaller value of δ.

At the beginning, the decay processes are different. In the source room, the decay starts approximately as:

$$E_1 \approx E_{I1}(1-2\delta_I t)+E_{II1}(1-2\delta_{II} t)$$

That is, it begins as if with the damping constant:

$$\delta_{01}=\frac{E_{I1}\delta_I+E_{II1}\delta_{II}}{E_{I1}+E_{II1}} \tag{3.39}$$

For the second room, we would get the corresponding apparent damping constant:

$$\delta_{02}=\frac{E_{I2}\delta_I+E_{II2}\delta_{II}}{E_{I2}+E_{II2}} \tag{3.40}$$

Only if the initial values E_{01} and E_{02} in the two rooms fulfil the coupling equations, (3.22) and (3.23), with $\delta=\delta_I$ or $\delta=\delta_{II}$ (it would be possible *only* with those eigen-values), could we get the same purely exponential decay in both rooms. For *arbitrary* initial values of E_{01} and E_{02}, the pairs E_{I1}, E_{I2} and E_{II1}, E_{II2} must fulfil these conditions separately, for δ_I or δ_{II}, respectively. Therefore, we may eliminate two of these quantities: it appears reasonable to drop E_{I2} and E_{II1}, since they appear to be quantities introduced on account of the coupling. Furthermore, it is expedient in each case to use the condition that avoids the small differences in $|\delta_I-\delta_1|$ or $|\delta_{II}-\delta_2|$. Thus, we get, from eqn. (3.23):

$$E_{I2}=E_{I1}\frac{\frac{c}{4}\tau S_{12}}{\frac{c}{4}A_{22}-2\delta_I V_2}=E_{I1}\frac{k_2}{1-\delta_I/\delta_2} \tag{3.41}$$

and by (3.22):

$$E_{II1}=E_{II2}\frac{k_1}{1-\delta_{II}/\delta_1} \tag{3.42}$$

Since $\delta_I<\delta_2$ always implies $\delta_{II}>\delta_1$, and $\delta_I>\delta_2$ always implies $\delta_{II}<\delta_1$, these equations show that if E_{I2} and E_{I1} have the same sign, then E_{II1} and E_{II2} must have different signs. Therefore, we get the equations for the resulting reverberation processes by putting eqns. (3.41) and (3.42) into (3.37) and (3.38):

$$E_1 = E_{I1} e^{-2\delta_I t} + E_{II2} \frac{k_1}{1 - \delta_{II}/\delta_1} e^{-2\delta_{II} t} \tag{3.43}$$

$$E_2 = E_{I1} \frac{k_2}{1 - \delta_I/\delta_2} e^{-2\delta_I t} + E_{II2} e^{-2\delta_{II} t} \tag{3.44}$$

That is, whatever the signs of E_{I1} and E_{II2} may be, we get in one room the sum, and in the other room the difference, of the elementary exponential processes. Therefore, the final purely exponential process is asymptotically approached both from above and below.

We get E_{I1} and E_{II2} from the initial values E_{01} and E_{02} by the conditions:

$$E_{01} = E_{I1} + E_{II2} \frac{k_1}{1 - \delta_{II}/\delta_1}$$

$$E_{02} = E_{I1} \frac{k_2}{1 - \delta_I/\delta_2} + E_{II2}$$

with the following results:

$$E_{I1} = \frac{E_{01} - E_{02} k_1/(1 - \delta_{II}/\delta_1)}{1 - \kappa^2/(1 - \delta_I/\delta_2)(1 - \delta_{II}/\delta_1)} \tag{3.45}$$

$$E_{II2} = \frac{E_{02} - E_{01} k_2/(1 - \delta_I/\delta_2)}{1 - \kappa^2/(1 - \delta_I/\delta_2)(1 - \delta_{II}/\delta_1)} \tag{3.46}$$

The differences expressed in the numerators show that it is possible for the ratio of the initial energy densities, E_{01}/E_{02}, to be chosen so that one of the decay types vanishes.

If we introduce eqns. (3.45) and (3.46) into (3.43) and (3.44), then we can find the general solution for given values of E_{10} and E_{20}. But we are again interested only in the special solutions where a sound source radiates the power P_1 in the room 1.

Here we assume, first, that the reverberation follows a sufficiently long steady-state excitation that we can assume a stationary energy density, as we discussed in Sections II. 3.1 and II. 3.2. Thus, we can make use of all the approximations in those sections, for small κ and different damping constants, according to eqn. (3.31). Then we may assume:

$$E_{01}=\frac{4P_1}{cA_{11}} \qquad E_{02}=k_2E_{01}$$
$$E_{11}=E_{01} \qquad E_{112}=-k_2\frac{\delta_1}{\delta_2-\delta_1}E_{01} \quad \text{etc.}$$

Therefore, we can describe the reverberation processes by:

$$E_1 \approx \frac{4P_1}{cA_{11}}\left[e^{-2\delta_1 t}+\kappa^2\frac{\delta_1{}^2}{(\delta_2-\delta_1)^2}e^{-2\delta_2 t}\right] \tag{3.47}$$

$$E_2 \approx \frac{4P_1}{cA_{11}}k_2\left[\frac{\delta_2}{\delta_2-\delta_1}e^{-2\delta_1 t}-\frac{\delta_1}{\delta_2-\delta_1}e^{-2\delta_2 t}\right] \tag{3.48}$$

We gather from these equations that in the source room the second term (which is produced by the coupling) is rather small at the beginning, so that it has no influence on the initial level and the first portion of the decay. Only later does it become predominant on account of the slower decay. In the neighboring room, by contrast, we must take into account both terms from the beginning; and the second term exceeds the first very soon.

Furthermore, we see that, whatever the values for δ_1 and δ_2 may be, the first and second terms are always added for the source room; this means that in that room the asymptotic decay is always approached from above: therefore, in the neighboring room it is always approached from below. It is plausible that the initial stationary level in the source room is higher than that which would correspond to the second decay-type only.

From eqn. (3.40) we can deduce, making use of all the approximations, that the initial damping constant δ_{02} vanishes, so that the decay $E_{2(t)}$ in the neighboring room starts with a horizontal tangent (zero slope). This holds not only for small κ and different values for δ_1 and δ_2; it follows physically from the general initial conditions. When the source is stopped in room 1, only dE_1/dt but not E_1, can suddenly change. In the neighboring room, the quantity $(c/4\ \tau S_{12}E_1)$ represents the power introduced there from the source room. But since that quantity cannot change suddenly, it follows that the decay $E_{(t)}$ can begin only with $dE_2/dt=0$.

We must now distinguish between the two extreme cases where $\delta_1 \gg \delta_2$ and $\delta_1 \ll \delta_2$; that is, where the source room is highly damped and the neighboring room is reverberant, or vice versa.

The first case may occur when we open the door from a living room, richly furnished with carpet and luxurious upholstery, into a bare entrance hall. If we shout in the living room, we hear first the reverberation with short decay of the living room; but when the sound level there has decreased sufficiently, we then notice only the longer reverberation of the entrance hall. The corresponding diagram of level versus time is plotted as a full line in Fig. 3.4, left. The dashed line corresponds to the decay of the level in the entrance hall; notice in this case the horizontal tangent at the beginning of the decay.

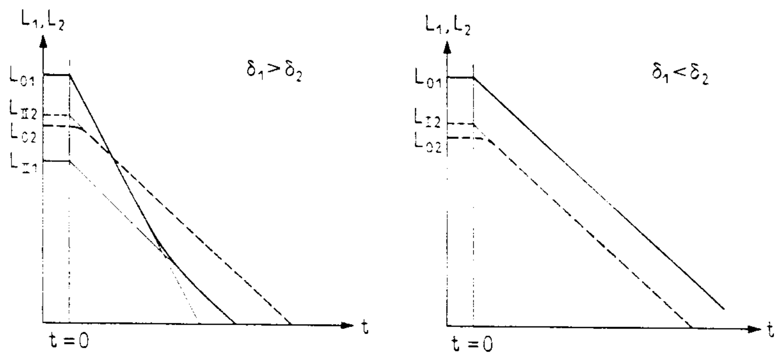

Fig. 3.4. Curves of sound level versus time for reverberation processes in coupled rooms, after stopping a steady-state source in room 1. Left: the source room is highly damped and the adjacent room is reverberant. Right: the adjacent room is highly damped and the source room is reverberant. Room 1———, room 2 – – –.

The other extreme case, $\delta_1 \ll \delta_2$, corresponds to our initial problem; that is, to theater boxes, seating areas under balconies, and side aisles connected to a high nave; all of these are relatively small, 'dead' spaces coupled to a larger 'live' space where the source of sound is located.

Again, Fig. 3.4 (right) shows the curves of level versus time; the full line corresponds to the source room, the dashed line to the neighboring room. In the source room, since even at the beginning of the decay its own longer reverberation process predominates, the shorter decay of the neighboring room is not heard at all. In the neighboring rooms, however, we find a process similar to that described above. The decay starts with a horizontal tangent and then diminishes according to the continuing but declining supply of energy from the source room. In practice, this first part of the decay is not apparent, since the necessary assumption of the statistical theory (i.e. a homogeneous, isotropic sound field) is not established until after many sound reflections. But the shift of the decay slope by a certain delay may be observed.

As a second example, we will discuss the reverberation process that follows an impulsive supply of energy $P_1\Delta t$ in the source room. Here we can make use of the relations with steady-state excitation already mentioned in Section II. 1.3. We have only to replace the quantity E_- in eqn. (1.18) with the expressions for E_1 and E_2 given in (3.47) and (3.48). We immediately find, for the reverberation processes following an impulsive excitation:

$$E_1' = \frac{4P_1\Delta t}{cA_{11}}\left(2\delta_1 e^{-2\delta_1 t} + \kappa^2 \frac{2\delta_1^2\delta_2}{(\delta_2-\delta_1)^2} e^{-2\delta_2 t}\right)$$

$$= \frac{P_1\Delta t}{V_1}\left(e^{-2\delta_1 t} + \kappa^2 \frac{\delta_1\delta_2}{(\delta_1-\delta_2)^2} e^{-2\delta_2 t}\right) \quad (3.49)$$

$$E_2' = \frac{P_1\Delta t}{V_1} k_2 \frac{\delta_2}{\delta_2-\delta_1}\left(e^{-2\delta_1 t} - e^{-2\delta_2 t}\right) \quad (3.50)$$

Figure 3.5 shows the corresponding curves of level versus time, similar to those of Fig. 3.4. Especially striking is the difference with respect to E_2, which starts at zero with ($\log E_2 \to -\infty$). This corresponds to the system equations. (3.18) and (3.19), assuming that we add an impulsive source in room 1. During the impulse, only the derivatives of highest

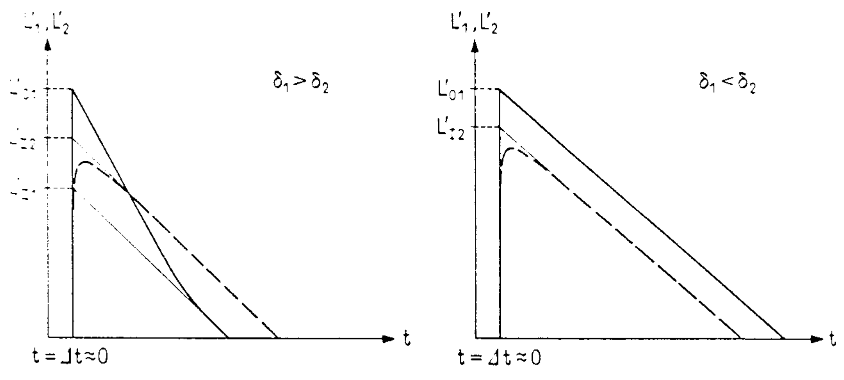

Fig. 3.5. Curves of level versus time as in Fig. 3.4, but with impulsive excitation of room 1. Left: source room more highly damped. Right: source room more reverberant than the neighboring room. Room 1———, room 2 - - -.

order (here dE/dt) play a role. But this means that E_2 does not change at all during the impulse, so it starts with the initial value:

$$E_{20}' = 0, \tag{3.51}$$

whereas E_1 jumps from zero to:

$$E_{10}' = \frac{1}{V_1}\int_{\Delta t} P_1 \, dt = \frac{\bar{P}_1 \Delta t}{V_1} \tag{3.52}$$

This corresponds to the same equal distribution of the supplied energy over the volume V_1 as we assumed in Section I. 1.3 for the volume V of a single room. In the present case, we cannot actually expect that this initial condition will be reached during the impulse duration: but it is reasonable to assume that this analysis describes fairly well the later part of the reverberation process.

We could have derived eqns. (3.49) and (3.50) in an alternative way by putting the initial values E_{20}' and E_{10}', given by eqns. (3.51) and (3.52), into (3.45) and (3.46) and considering the same simplifying approximations that we used earlier in (3.47) and (3.48).

Figure 3.5 (left) also shows that in the case $\delta_1 \gg \delta_2$ the reverberation in room 1 has changed: it is not a change in principle, but (at the right) in the level at which the bend in the curve occurs. Since L_{III} is decreased by $10 \log (\delta_1/\delta_2)$, the level of the bend in the curve is correspondingly lowered. In practice, the bend may not appear in the part of the curve above the background noise that limits the recording. In the case of coupled rooms, it becomes very important to distinguish whether the reverberation follows steady-state or impulsive excitation, and, in the latter case, whether the reverberation was determined by direct recording or by 'backward-integration' (see Section II. 4.5) (which would give results corresponding to steady-state excitation).

A typical case of coupled rooms occurs in opera houses. The reverberation time of the auditorium is usually low (1·2 up to (at most) 1·6 s) because of the custom of fitting the greatest possible number of seats into the given room volume. On the performer's side of the proscenium, however, the stage house may have a reverberation time up to 3 s. It is paradoxical that open-air scenes, played on a bare stage, are given the most reverberant environment! In such cases, the sound events on the stage may produce in the auditorium slopes such as those in Fig. 3.4 and 3.5, right (the dashed lines). On the other hand, the pit orchestra, being located more in the auditorium, may produce curves like the solid lines in the left parts of the figures.

Such a difference, in fact, may even be attractive up to a certain point, so that experienced listeners just notice it, but others in the audience perceive it as a 'special' but undefined quality.

There have been occasional attempts to compensate for a too-short reverberation time in the auditorium by allowing the audience to hear the longer reverberation time of the stage house; but we have learned in this section that the decay process thus achieved is quite different from that which a longer reverberation time in the auditorium would give.

II.3.5 Electroacoustical Coupling between Rooms

Any consideration of coupled rooms would be incomplete today if we took into account only the 'natural' coupling, discussed up to this point, without also accounting for the possibility of 'artificial' coupling by means of microphones, amplifiers and loudspeakers.

The most frequently used applications of the latter occur with radio or television broadcasts and in cinema theaters. The sound is picked up by a microphone in one room and is transmitted, either 'live' or recorded, into another room (a living room or a cinema theater), where it is re-radiated by loudspeakers.

This kind of coupling exhibits two essential differences from natural coupling. First, the coupling is uni-directional; that is, with respect to our energy balance, room 1 transfers into room 2 an amount of power P_{21} that is proportional to E_1:

$$P_{21} = K_{21} E_1 \tag{3.53}$$

but room 2 transmits no power back to room 1 at all.

The second difference is that the power radiated into room 2 is not subtracted from the power P_1 radiated into room 1; that is, the electroacoustical equipment does not absorb sound energy in room 1 and thus has no influence on the value of A_1. (This is not as self-evident as it appears. At least we will discuss later an arrangement by which the electroacoustical equipment actually adds to P_1 and thereby reduces the value of A_1.)

The power P_{21} is taken from an independent power source, which is 'piloted' by the time-varying E_1; only in this manner is the uni-directional coupling possible. The constant K_{21}, which contains the gain